RAMAIAH
Institute of Technology

MATLAB

Lab Manual

Differential Equations and Laplace Transforms – (MAE21)

BE II Semester, Electronics and Electrical Engineering Stream

Department of Mathematics, Ramaiah Institute of Technology, Bengaluru - 54

Lab 01

Taylor series for Single Variable:

Syntax

```
taylor(f,var)
taylor(f,var,a)
```

approximates `f` with the Taylor series expansion of `f` at the point `var = a`.

Program 01

```
syms x
T1=taylor(exp(x)) %Maculurian Series
T2=taylor(sin(x))
T3=taylor(log(sin(x)),x,2) % Taylor series at x=2
T4=taylor(log(x),x,1)
```

Output:

```
T1 =
 x^5/120 + x^4/24 + x^3/6 + x^2/2 + x + 1

T2 =
 x^5/120 - x^3/6 + x

T3 =
log(sin(2)) - (x - 2)^4*(cos(2)^2/(12*sin(2)^2) + (cos(2)*(cos(2)/(12*sin(2))
+ (cos(2)*(cos(2)^2/(4*sin(2)^2) + 1/6))/sin(2)))/sin(2) + 1/12) + (x -
2)^3*(cos(2)/(12*sin(2)) + (cos(2)*(cos(2)^2/(3*sin(2)^2) + 1/4))/sin(2)) +
(x - 2)^5*((7*cos(2))/(240*sin(2)) + (cos(2)*((5*cos(2)^2)/(72*sin(2)^2) +
(cos(2)*((5*cos(2))/(72*sin(2)) + (cos(2)*(cos(2)^2/(5*sin(2)^2) +
1/8))/sin(2)))/sin(2) + 1/16))/sin(2) - (cos(2)*(cos(2)^2/(3*sin(2)^2) +
1/4))/(6*sin(2)) + (cos(2)*(cos(2)^2/(4*sin(2)^2) + 1/6))/(2*sin(2))) -
(cos(2)^2/(2*sin(2)^2) + 1/2)*(x - 2)^2 + (cos(2)*(x - 2))/sin(2)

T4 =
x - (x - 1)^2/2 + (x - 1)^3/3 - (x - 1)^4/4 + (x - 1)^5/5 - 1
```

Program 02

```
syms x
T1=taylor(exp(x)) %Maculurian Series
T2=taylor(sin(x))
T3=taylor(log(sin(x)),x,2) % Taylor series at x=2
T4=taylor(log(x),x,1)
```

Output:

```
T1 =
 x^5/120 + x^4/24 + x^3/6 + x^2/2 + x + 1

T2 =
 x^5/120 - x^3/6 + x

T3 =
log(sin(2)) - (x - 2)^4*(cos(2)^2/(12*sin(2)^2) + (cos(2)*(cos(2)/(12*sin(2))
+ (cos(2)*(cos(2)^2/(4*sin(2)^2) + 1/6))/sin(2)))/sin(2) + 1/12) + (x -
2)^3*(cos(2)/(12*sin(2)) + (cos(2)*(cos(2)^2/(3*sin(2)^2) + 1/4))/sin(2)) +
(x - 2)^5*((7*cos(2))/(240*sin(2)) + (cos(2)*((5*cos(2)^2)/(72*sin(2)^2) +
```

```
(cos(2)*((5*cos(2))/(72*sin(2)) + (cos(2)*(cos(2)^2/(5*sin(2)^2) +
1/8))/sin(2)))/sin(2) + 1/16))/sin(2) - (cos(2)*(cos(2)^2/(3*sin(2)^2) +
1/4))/(6*sin(2)) + (cos(2)*(cos(2)^2/(4*sin(2)^2) + 1/6))/(2*sin(2))) -
(cos(2)^2/(2*sin(2)^2) + 1/2)*(x - 2)^2 + (cos(2)*(x - 2))/sin(2)

T4 =
x - (x - 1)^2/2 + (x - 1)^3/3 - (x - 1)^4/4 + (x - 1)^5/5 - 1
```

To represent term in ascending order:

Syntax

```
sympref('PolynomialDisplayStyle','ascend');
```

Program

```
sympref('PolynomialDisplayStyle','ascend');
T1
```

Output:

```
T1 =
1 + x + x^2/2 + x^3/6 + x^4/24 + x^5/120
```

Taylor series for Two-Variable:

Program

```
syms x y
f = y*exp(x - 1) - x*log(y);
T = taylor (f, [x y], [1 1],'Order',3) % Represent # of term in the series
```

Output

```
T =
x + (- 1 + x)^2/2 + (- 1 + y)^2/2
```

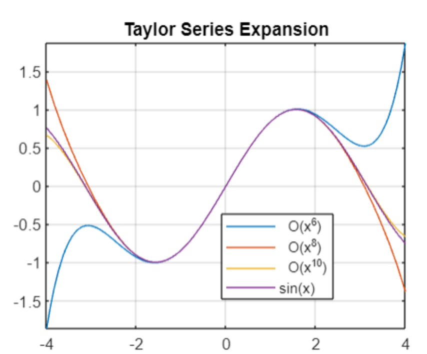
Taylor series for multvariable:

```
syms x y z
f = sin(x) + cos(y) + exp(z);
T = taylor(f)
```

Output

```
T =
x + cos(y) + exp(z) - x^3/6 + x^5/120
```

Plotting Taylor series for different order	**Output**

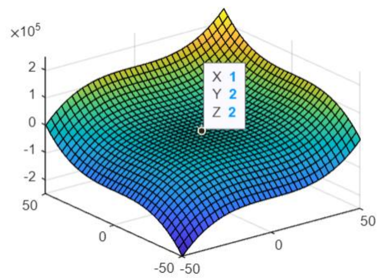
`syms x`
`f = sin(x);`
`T6 = taylor (f, x);`
`T8 = taylor (f, x,'Order',8);`
`T10 = taylor (f, x,'Order',10);`
`fplot ([T6 T8 T10 f])`
`xlim ([-4 4])`
`grid on`
`legend(O(x^6)', O(x^8)', O(x^{10})',`
`'sin(x)')`
`title('Taylor Series Expansion')`
	

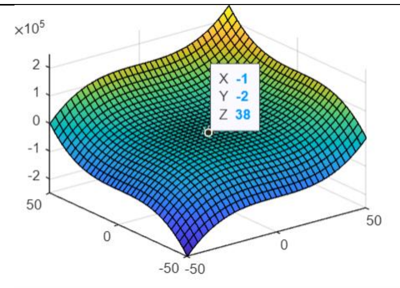
Maxima and Minima for two variable functions:

```
syms x y real
f=input('Enter the function f(x,y):')

dfx=diff(f,x); %first derivative
dfy=diff(f,y);

eqns=[dfx==0,dfy==0];
S=solve(eqns,[x y], 'Real', true);% Gives stationary points
G=S.x;
H=S.y;

dfxx=diff(dfx,x); % f_xx %double derivative
dfyy=diff(dfy,y); % f_yy
dfxy=diff(dfy,x); %f_xy

A=subs(dfxx,S); %for AC-B^2
B=subs(dfxy,S);
C=subs(dfyy,S);
fun=subs(f,S); % Subsitutuion of values

for i=1: length(A)

if A(i)*C(i)-B(i)*B(i)>0 & A(i)<0
 fprintf('The Maximum point is %d %d',[G(i), H(i)]);
 fprintf('\n The maximum value is %d',fun(i));
figure;
fsurf(f,[-3 3 -3 3]);
 hold on
 plot3(G(i),H(i),fun,'*w','markersize',10)
hold off

 elseif A(i)*C(i)-B(i)*B(i)>0 & A(i)>0
 fprintf('Minimum point is %d %d',[G(i), H(i)]);
 fprintf('\n The minimum value is %d',fun(i));
figure;
fsurf(f,[-3 3 -3 3]);
 hold on
 plot3(G(i),H(i),fun,'*w','markersize',10)
hold off

elseif A(i)*C(i)-B(i)*B(i)<0
 fprintf('Saddle point is %d %d', [G(i), H(i)]);

else
 fprintf('No conclusion can be drawn for %d %d',[G(i), H(i)]);
end
end
```

Output:

```
Enter the function f(x,y):
x^3+y^3-3*x-12*y+20
The Maximum point is -1 -2
The maximum value is 38
Saddle point is 1 -2
Saddle point is -1 2
Minimum point is 1 2
The minimum value is 2
```





Exercise Problems

1. Using Maclaurin's series expand $\sqrt{1+\sin x}$ up to the term containing x^4.
2. Obtain the first four terms of Taylor's series of $\cos x$ about $x=\frac{\pi}{3}$.
3. Expand $\sin^{-1} x$ in powers of x up to second-degree term.
4. Expand a^x in powers of x up to the first three terms.
5. Expand $\tan^{-1} x$ in powers of $(x-1)$ up to the term containing $(x-1)^4$.
6. Expand $\log(1+\sin 2x)$ in powers of x up to the term containing x^4.
7. Expand the following functions in powers of x and y up to second-degree terms:

   i) $\sin x \sin y$ ii) $e^x \sin y$ iii) $e^{x^2-y^2}$ iv) $e^x \log(1+y)$.
8. Expand the following functions at the given point up to second-degree terms:

   i) $xy^2+\cos(xy)$ about $\left(1, \pi/2\right)$ ii) $x^2y+3y-2$ about (1,-2) iii) x^y about (1,1).
9. Discuss the maxima and minima of

   (i) $x^3y^2(1-x-y)$. (ii) x^3+y^3-3axy (iii) $f(x,y)=xy(1-x-y)$.

Lab 02

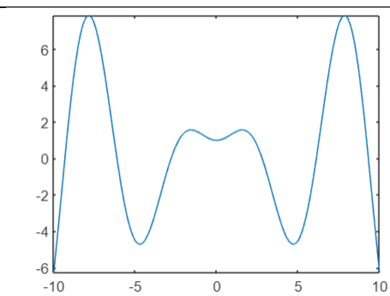
Newton-Raphson method:

Single Variable:

```
clc;
clear;
syms x
f(x)= x*sin(x)+cos(x);
df(x)= diff (f, x);
a=2; b=3;
if f(a)*f(b)<0
x0=(a+b)/2;
else
fprintf ("change interval value")
end
 %x0=input ("enter the initial approximation:");
for n=1: 5000
    xn= x0-(f(x0)/df(x0));
fprintf ('\n The required root at %d is: %f', n, xn);
if abs(f(xn)) <=0.000001
      break;
    else
        x0=xn;
    end
end
fplot(f);
xlim([-10 10])
```

Output:

```
The required root at 1 is: 2.847022
The required root at 2 is: 2.799175
The required root at 3 is: 2.798386
```



Multivariable:

```
clc;
syms x y h k
f (x, y) =x^2-y^2-4;
g (x, y) =y^2+x^2-16;

dfx = diff (f, x);%derivative
dfy = diff(f,y);
dgx = diff(g,x);
dgy = diff(g,y);

 x0=input ("Enter the initial value of x:");
 y0=input ("Enter the initial value of y:");

 for i=1:1000
           EQ1=dfx (x0, y0) *h+dfy (x0, y0) *k+f (x0, y0);
           EQ2=dgx (x0, y0) *h+dgy (x0, y0) *k+g (x0, y0);
```

```
            S=solve ([EQ1==0, EQ2==0],[h,k]);
            xn=x0+S.h;
            yn=y0+S.k;
fprintf ("The required root at %d is %f \n %f\n",i,xn,yn);
    if abs (f(xn, yn))<=0.00001
            break;
            else
                x0=xn;
                y0=yn;
    end
 end
```

Output:

```
Enter the initial value of x:1
Enter the initial value of y:1
The required root at 1 is 5.500000 3.500000
The required root at 2 is 3.659091 2.607143
The required root at 3 is 3.196005 2.454256
The required root at 4 is 3.162456 2.449494
The required root at 5 is 3.162278 2.449490
```

Exercise Problems

1. Using Newton – Raphson iterative method, find a real root of the following equations correct to 4 places of decimals.
   (i) $x^4 - 12x + 7 = 0$ (ii) $x + \log_{10} x = 3.375$ (iii) $2 \tan x = 3x$ (iv) $3x - \cos x - 1 = 0$
   (v) $x \sin x + \cos x = 0$ between 2 and 3 (vi) $cosx = x^2$ $near$ 1
   (vii) tanx + tanhx = 0 near x = 2.5 (viii) $x^3 - 2x + 0.5 = 0$ (ix)$x \log_{10} x = 1.2$ near x = 2.5
2. Using Newton – Raphson iterative method, find a real root of the following equations correct to 4 places of decimals.
   i) $x^2 + y = 11; y^2 + x = 7$ $(x_0, y_0) = (3.5, -1.8)$
   ii) $x^3 = y + 100$; $y^3 = x + 150$ $(x_0, y_0) = (4.5,5.3)$
   iii) $x^2 = 3xy - 7;\ y = 2(x + 1)$ $(x_0, y_0) = (0.7,3.4)$
   iv) $x^2 + y^2 = 4;\ 4x^2 - y^2 = 4$ $(x_0, y_0) = (1,1)$

Lab 03

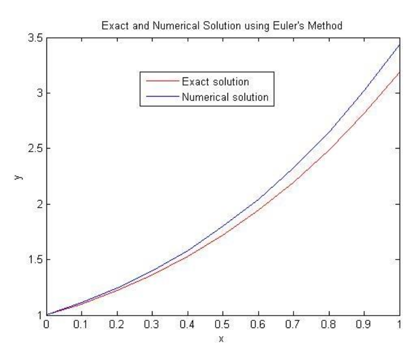
Numerical Solution to ODE

Euler's method

```
%Program to solve ordinary differential equations using Euler's method
clc;
close;
clear;
% Define the function f(x,y) that represents the differential equation
f = @(x,y) x + y;
% Define the initial conditions
x0 = 0; % initial x value
y0 = 1; % initial y value
xn = 1; % final x value
% Define the step size and the range of x values to approximate
h = 0.1; % step size
x = x0:h:xn; % range of x values
% Initialize the y vector with the initial value y0
y = zeros(size(x));
y(1) = y0;
% Use Euler's method to approximate the solution
for i = 2:length(x)
    y(i) = y(i-1) + h * f(x(i-1), y(i-1));
    disp(y(i))
end
%Analytical solution
syms s(t)
ode = diff(s)-t-s;
cond = s(0) == 1;
sSol(t) = dsolve(ode,cond)
t=[0:0.1:1];
%To  plot numerical solution
 plot(x, y, 'r');
 hold on
%To plot the approximate solution
plot(t, sSol(t),'b')
hold off
legend('Exact solution','Numerical solution')
xlabel('x');
ylabel('y');
title('Exact and Numerical Solution using Euler''s Method');
```

Output

```
     1.1000
     1.2200
     1.3620
     1.5282
     1.7210
     1.9431
     2.1974
     2.4872
     2.8159
     3.1875
 sSol(t) =
 2*exp(t) - t - 1
```



Modified Euler’s method

```
%Program to solve ordinary differential equations using modified Euler’s method
clc;
clear;
close;
x0 = 0;
y0 = 1;
h = 0.1; % Step size
xn = 0.1; % Final value
% Define the function y' = f(x,y)
f = @(x,y) -y^2+ x ;
 % Implement Euler's modified method
x = x0:h:xn;
y = zeros(size(x));
y(1) = y0;
 for i = 2:length(x)
    % Predictor step
    y(i) = y(i-1) + h*f(x(i-1), y(i-1));
    fprintf('Required value by Euler''s method (Predictor method) %f \n :' , y(i));
    %disp(y(i))
    x(i)=x(i-1)+h;
    y(i,1)=y(i);
    % Corrector step
    for j=2:20
    y(i,j) = y(i-1) + (h/2)*(f(x(i-1), y(i-1)) + f(x(i), y(i,j-1)));
   disp(y(i,j));
    if abs(y(i,j)-y(i,j-1))<0.00001
        y(i)=y(i,j);
        fprintf('Required value by Euler''s modified method (Corrector method) %f \n :' , y(i));
        break;
    end
    end
end
```

Output:

```
Required value by Euler's method (Predictor method) 0.900000 :
0.9000
0.9145
0.9132
0.9133
0.9133
0.9133
Required value by Euler's modified method (Corrector method) 0.913295
```

Lab 04

Taylor's method

```
%Program to solve ordinary differential equations using modified Taylor's
method
clc;
clear;
syms x y;
%Define function
f(x,y)=1/(x^2+y);
y0=4;
h=0.1;
x0=4;
xn=4.1;
% Define the order of the Taylor series expansion
n=1;
y(1)=y0;
y(2)=f(x,y);
% Compute the values of h^n/n! for n = 0, 1, 2, 3,...,
ht=h.^(0:n)./factorial(0:n);
for i=2:n
    % To compute the values of derivatives
            y(i+1)=diff(y(i),x)
end
for i=1:n
    x=x0+i*h;
    c=eval(y);
    % Compute the values of yb using the Taylor series expansion
    yb=sum(c.*ht);
    fprintf('The value of y(%f) is %f\n',x,yb);
end
```

Output

```
The value of y(4.100000) is 4.004805
```

Runge Kutta 4th order method

```
%Program to solve ordinary differential equations using Runge Kutta 4th order
method
clc;
clear;
f = @(x,y) x+y;
%Define the interval
x0=1;
y0=1;
h=0.2;
xn=2;
n=(xn-x0)/h;
x = x0:h:xn;
y = zeros(size(x));
y(1) = y0;
x(1)=x0;
 for i = 2:length(x)
    k1=h*f(x(i-1),y(i-1));
k2=h*f(x(i-1)+h/2,y(i-1)+k1/2);
k3=h*f(x(i-1)+h/2,y(i-1)+k2/2);
k4=h*f(x(i-1)+h,y(i-1)+k3);
      y(i)=y(i-1)+(k1+2*k2+2*k3+k4)/6;
      x(i)=x(i-1)+h;
    fprintf("The value of y(%f) is %f\n", x(i), y(i));
```

```
end
```

Output

```
The value of y(1.200000) is 1.464200
The value of y(1.400000) is 2.075454
The value of y(1.600000) is 2.866319
The value of y(1.800000) is 3.876562
The value of y(2.000000) is 5.154753
```

Exercise Problems

1. Solve the following initial value problems by Euler's method.
   a) $\frac{dy}{dx} = \frac{y-x}{y+x}$; $y(0) = 1$ at $x = 0.4$ by taking $h = 0.1$
   b) $\frac{dy}{dx} = x \log y - y \log x$; $y(1) = 1$ at $x = 1.4$ by taking $h = 0.1$
2. Solve the following initial value problems using Modified Euler's method (Carry out three iterations at each stage)
   a) $\frac{dy}{dx} = \frac{2y}{x} + x^3$; $y(1) = 0.5$ at $x = 1.4$ by taking $h = 0.2$
   b) $\frac{dy}{dx} = x + y^2$; $y(0) = 1$ at $x = 0.2$ by taking $h = 0.1$
3. Solve the following initial value problems using Runge - Kutta method of fourth order
   a) $\frac{dy}{dx} = y - x^2$; $y(0.6) = 1.7379$ at $x = 0.8$ by taking $h = 0.1$
   b) $\frac{dy}{dx} = xy + y^2$; $y(0) = 1$ at $x = 0.2$ by taking $h = 0.2$
4. Solve the following initial value problems by Taylor's series method by considering terms up to fourth degree
   a) $\frac{dy}{dx} = 1 - 2xy$; $y(0) = 0$ at $x = 0.2$
   b) $\frac{dy}{dx} = xy^2 - 1$; $y(0) = 1$ at $x = 0.1$

Lab 5

Solving Higher Order Ordinary Differential Equation

`dsolve(eqn)` solves the differential equation eqn, where eqn is a symbolic equation. Use `diff` and == to represent differential equations. For example, diff(y,x) == y represents the equation $dy/dx = y$. Solve a system of differential equations by specifying eqn as a vector of those equations.

```
clc;
clear all;
% Define the differential equation
syms y(x);
eqn=input(' Enter the differential equation')
yg= simplify( dsolve(eqn))
```

Problem 1:

$$\frac{d^3y}{dx^3} - 2\frac{d^2y}{dx^2} + 4\frac{dy}{dx} - 8y = 0$$

Output 1:

```
Enter the differential equation
diff(y,x,3) - 2*diff(y,x,2) + 4*diff(y,x)- 8*y == 0

eqn(x) =
4*diff(y(x), x) - 8*y(x) - 2*diff(y(x), x, x) + diff(y(x), x, x, x) == 0

yg =
C2*cos(2*x) + C1*exp(2*x) - C3*sin(2*x)
```

Problem 2:

$$\frac{d^2y}{dt^2} - \frac{dy}{dt} + 9y = 5e^{-2t}$$

Output 2:

```
Enter the differential equation
diff(y,x,2) - 6* diff(y,x)+ 9*y == 5*exp(-2*x)

eqn(x) =
9*y(x) - 6*diff(y(x), x) + diff(y(x), x, x) == 5*exp(-2*x)

yg =
exp(-2*x)/5 + C1*exp(3*x) + C2*x*exp(3*x)
```

Problem 3:

$$x^2\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} - 12y = 6x + 5$$

Output 2:

```
Enter the differential equation
x^2*diff(y,x,2)-2*x*diff(y,x,1) -12*y == 6*x+5

eqn(x) =
```

```
x^2*diff(y(x), x, x) - 12*y(x) - 2*x*diff(y(x), x) == 6*x + 5

yg =
C1*x^(57^(1/2)/2 + 3/2) - (3*x)/7 + C2*x^(3/2 - 57^(1/2)/2) - 5/12
```

Exercise Problems

1. $(D^4 + 2D^3 + D)y = 0$
2. $(D^3 + 2D^2 + D)y = x^2 e^{2x} + \sin^2 x$
3. $\frac{d^2 y}{dx^2} + \frac{1}{x}\frac{dy}{dx} = \frac{12 \log x}{x^2}$
4. $(3x+2)^2 y'' + 3(3x+2)y' - 36y = 8x^2 + 4x + 1$

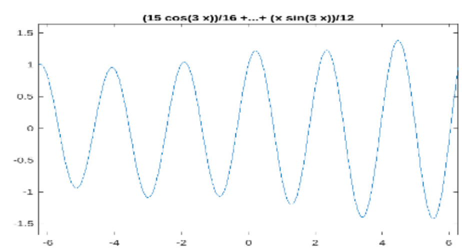
Solving higher order ordinary differential Equation with initial condition

```
clc;
clear all;
close all;
% Define the differential equation
syms y(x);
eqn=input('enter the differential equation')
Dy = diff(y,x);
cond=input('Enter the Initial Condition')
yg= simplify( dsolve(eqn))
yp = simplify(dsolve(eqn,cond))
ezplot(yp)
```

Problem 1:
$y'' + 9y = \cos 2x . \cos x$ and given that $y(0) = 1$ and $y'(0) = 2$ and also plot the graph

Output 1:

```
enter the differential equation
diff(y,x,2) + 9*y == cos(2*x)*cos(x)

eqn(x) =
9*y(x) + diff(y(x), x, x) == cos(2*x)*cos(x)

Enter the Initial Condition
 [y(0)==1, Dy(0)==2]
cond =
[y(0) == 1, subs(diff(y(x), x), x, 0) == 2]
yg =
(7*cos(3*x))/144 + cos(x)/16 + (x*sin(3*x))/12 + C1*cos(3*x) - C2*sin(3*x)
yp =
(15*cos(3*x))/16 + (2*sin(3*x))/3 + cos(x)/16 + (x*sin(3*x))/12
```

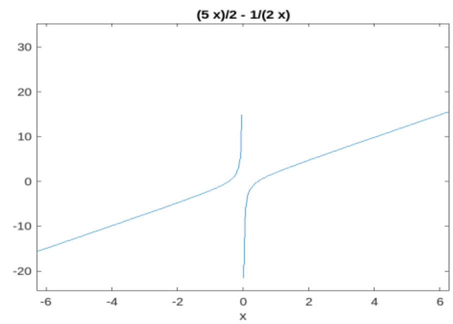


Problem 2:

$x^2 y'' + xy' - y = 0$ and given that $y(1) = 2$ and $y'(1) = 3$ and also plot the graph

Output 2:

```
enter the differential equation
x^2*diff(y,x,2)+x*diff(y,x,1)-y == 0

eqn(x) =
x^2*diff(y(x), x, x) - y(x) + x*diff(y(x), x) == 0

Enter the Initial Condition
 [y(1)==2, Dy(1)==3]

cond =
[y(1) == 2, subs(diff(y(x), x), x, 1) == 3]

yg =
C2*x + C1/(2*x)

yp =
(5*x)/2 - 1/(2*x)
```



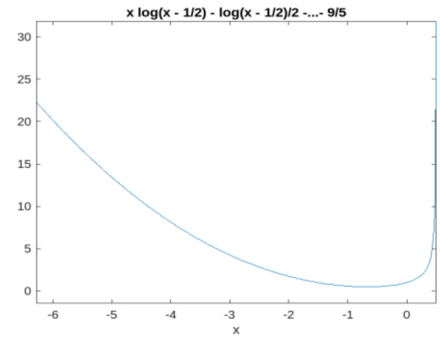
Problem 3:

$(2t-1)^2 y'' + (2t-1)y' - 2y = 8t^2 - 2t + 3$ and given that $y(0) = 1$ and $y'(0) = 2$ and also plot the graph

Output 3:

```
enter the differential equation
(2*x-1)^2*diff(y,x,2)+(2*x-1)*diff(y,x,1)-2*y == 8*x^2-2*x+3

eqn(x) =
(2*x - 1)^2*diff(y(x), x, x) - 2*y(x) + (2*x - 1)*diff(y(x), x) == 8*x^2 -
2*x + 3

Enter the Initial Condition
[y(0)==1, Dy(0)==2]

cond =
[y(0) == 1, subs(diff(y(x), x), x, 0) == 2]


yg =
x*log(x - 1/2) - log(x - 1/2)/2 - (4*x)/5 + C1*(x - 1/2) + (4*x^2)/5 -
(2*2^(1/2)*C2)/(3*(2*x - 1)^(1/2)) - 9/5


yp =
```

```
x*log(x - 1/2) - log(x - 1/2)/2 - (x - 1/2)*(2/3 - log(2) + pi*1i) - (4*x)/5
+ 37i/(15*(2*x - 1)^(1/2)) + (4*x^2)/5 - 9/5
```



Exercise Problems:

1. $(D^2 + D)y = 2 + 2x + x^2,\ y(0) = 8, y'(0) = -1$
2. $(D^3 + 2D^2 + D)y = x^2 e^{2x} + \sin^2 x$ and $y(1) = 0, y'(2) = \text{-}1$
3. $x^3 \dfrac{d^3y}{dx^3} + 3x^2 \dfrac{d^2y}{dx^2} + x\dfrac{dy}{dx} + y = x\log x$ and $y(2) = 0, y'(2) = \text{-}1$
4. $(3t+1)^2 y'' + (3t+1)y' - 2y = 8t^2 - 2t + 3$ and $y(1) = 0, y'(1) = \text{-}1$

Solving higher order ordinary differential Equation with boundary condition

```
clc;
clear all;
close all;
% Define the differential equation
syms y(x);
eqn=input('enter the differential equation')
cond=input('Enter the Boundary Condition')
yg= simplify( dsolve(eqn))
yp = simplify(dsolve(eqn,cond))
ezplot(yp)
```

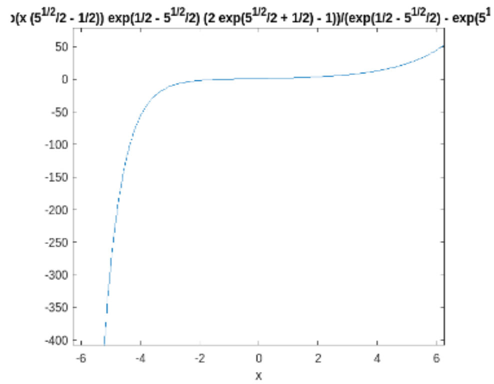
Problem 1:

$y'' = y$ with boundary conditions $y(0) = 1$ *and* $y(1) = 2$ and also plot the graph

Output 1:

```
enter the DE
diff(y(x), x) - y(x) + diff(y(x), x, x) == 0

eqn =
diff(y(x), x) - y(x) + diff(y(x), x, x) == 0

Enter the BC
 [y(0)==1, y(1)==2 ]

cond =
[y(0) == 1, y(1) == 2]

yg =
exp(-(x*(5^(1/2) + 1))/2)*(C1 + C2*exp(5^(1/2)*x))
```

```
yp =
- (exp(-x*(5^(1/2)/2 + 1/2))*(exp(5^(1/2)/2 + 1/2) - 2*exp(1/2 -
5^(1/2)/2)*exp(5^(1/2)/2 + 1/2)))/(exp(1/2 - 5^(1/2)/2) - exp(5^(1/2)/2 +
1/2)) - (exp(x*(5^(1/2)/2 - 1/2))*exp(1/2 - 5^(1/2)/2)*(2*exp(5^(1/2)/2 +
1/2) - 1))/(exp(1/2 - 5^(1/2)/2) - exp(5^(1/2)/2 + 1/2))
```

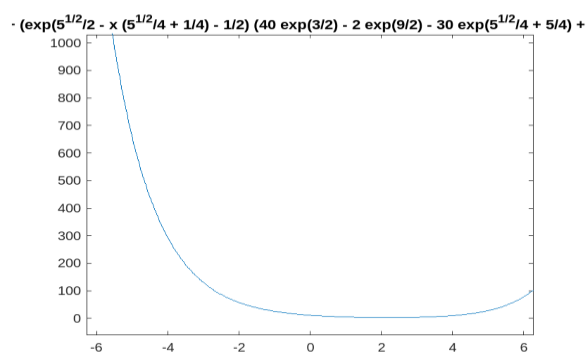


Problem 2:

$4y'' + 2y' - y = e^x$ with boundary conditions $y(2) = 3$ *and* $y(3) = 4$ and also plot the graph

Output 2:

```
enter the differential equation
 4*diff(y,x,2)+2*diff(y,x,1)-y == exp(x)

eqn(x) =
2*diff(y(x), x) - y(x) + 4*diff(y(x), x, x) == exp(x)

Enter the Boundary Condition
[y(2)==3, y(3)==4]

cond =
[y(2) == 3, y(3) == 4]

yg =
exp(x)/5 + C1*exp(-(x*(5^(1/2) + 1))/4) + C2*exp((x*(5^(1/2) - 1))/4)

yp =
exp((5*x)/4 + (5^(1/2)*x)/4 - x*(5^(1/2)/4 + 1/4))*(5^(1/2)/10 + 1/10) -
exp((5*x)/4 + (5^(1/2)*x)/4 - x*(5^(1/2)/4 + 1/4))*(5^(1/2)/10 - 1/10) -
(exp(x*(5^(1/2)/4 - 1/4) - 5^(1/2)/2 - 1/2)*(40*exp(3/2) - 2*exp(9/2) -
30*exp(5/4 - 5^(1/2)/4) + 2*exp(13/4 - 5^(1/2)/4)))/(10*(exp(1/4 - 5^(1/2)/4)
- exp(5^(1/2)/4 + 1/4))) + (exp(5^(1/2)/2 - x*(5^(1/2)/4 + 1/4) -
1/2)*(40*exp(3/2) - 2*exp(9/2) - 30*exp(5^(1/2)/4 + 5/4) + 2*exp(5^(1/2)/4 +
13/4)))/(10*(exp(1/4 - 5^(1/2)/4) - exp(5^(1/2)/4 + 1/4)))
```

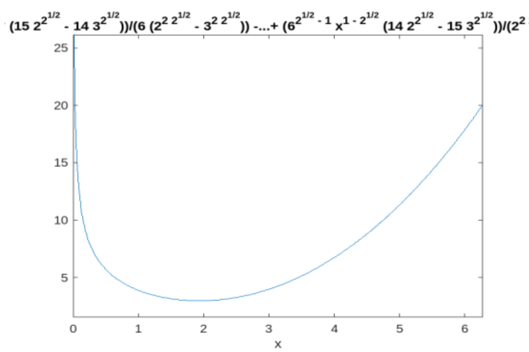


Problem 3:

$x^2 y'' - xy' - y = 2x$ with boundary conditions $y(2) = 3$ *and* $y(3) = 4$ and also plot the graph

Output 3:

```
enter the differential equation
x^2*diff(y,x,2)-x*diff(y,x,1)-y == 2*x

eqn(x) =
x^2*diff(y(x), x, x) - y(x) - x*diff(y(x), x) == 2*x

Enter the Boundary Condition
[y(2)==3, y(3)==4]

cond =
[y(2) == 3, y(3) == 4]

yg =
C1*x^(2^(1/2) + 1) - x + C2*x^(1 - 2^(1/2))

yp =
(x^(2^(1/2) + 1)*(15*2^(2^(1/2)) - 14*3^(2^(1/2))))/(6*(2^(2*2^(1/2)) -
3^(2*2^(1/2)))) - x + (6^(2^(1/2) - 1)*x^(1 - 2^(1/2))*(14*2^(2^(1/2)) -
15*3^(2^(1/2))))/(2^(2*2^(1/2)) - 3^(2*2^(1/2)))
```



Exercise Problems

1. $(D^2 + D)y = 2 - 2x - x^2,\ y(0) = 8, y(2) = -1$
2. $(D^3 + 2D^2 - D)y = x^2 e^{2x} - \sin^2 x$ and $y(1) = 0, \mathrm{y}(2) = -1$
3. $x^3 \dfrac{d^3 y}{dx^3} - 3x^2 \dfrac{d^2 y}{dx^2} - x\dfrac{dy}{dx} + y = x + \log x$ and $y(2) = 0, \mathrm{y}(2) = -1$
4. $(3t-1)^2 y'' + (3t-1)y' + 2y = 8t^2 + 2t + 3$ and $y(1) = 0, \mathrm{y}(1) = -1$

Lab 6

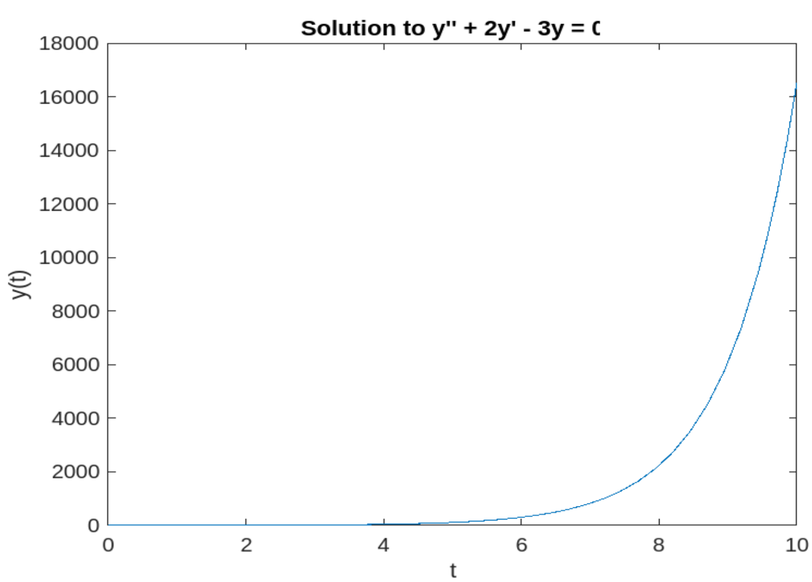
Solving Higher Order Ordinary Differential Equation Using ODE45 code

`[t,y] = ode45(odefun,tspan,y0)`, where `tspan = [t0 tf]`, integrates the system of differential equations `y'=f(t,y)` from `t0` to `tf` with initial conditions `y0`. Each row in the solution array y corresponds to a value returned in column vector `t`.

Program 1:

```
%Write a mat lab code for solving the differential equation  y'' + 2y' - 3y =
0 with initial conditions y(0) = 1 and y'(0) = 0 and also plot the graph
using ODE45 solver

clc;
eqn = @(t,y) [y(2); -2*y(2) + 3*y(1)];

% Define the initial conditions
t0 = 0;
y0 = [1; 0];

% Define the time interval for the solution
tspan = [0 10];

% Use the ODE45 solver to solve the differential equation
[t,y] = ode45(eqn, tspan, y0);

% Plot the solution
plot(t, y(:,1))
xlabel('t')
ylabel('y(t)')
title('Solution to y'''' + 2y'' - 3y = 0')
```

Output:

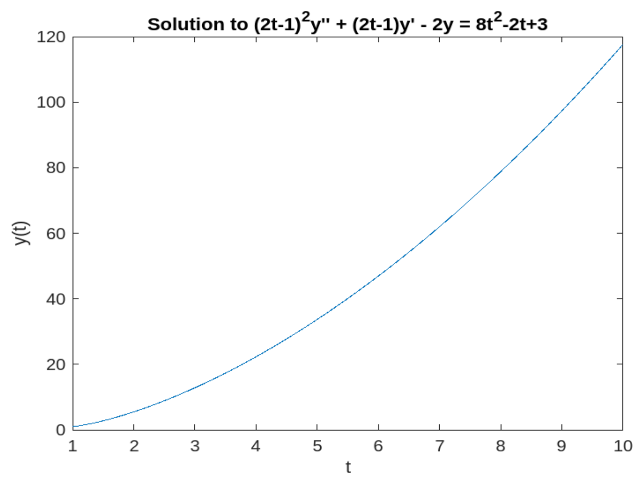


Program 2:

Write a mat lab code for solving the differential equation
$(2t-1)^2 y'' + (2t-1)y' - 2y = 8t^2 - 2t + 3$ with initial conditions y(0) = 1 and y'(0) = 2 and also plot the graph using ODE45 solver

```
clc;
        clear all;
        close all;
eqn = @(t,y) [y(2); (8*t^2-2*t+3-(2*t-1)*y(2)+2*y(1))/(2*t-1)^2];

% Define the initial conditions
t0 = 0;
y0 = [1; 2];

% Define the time interval for the solution
tspan = [1 10];

% Use the ODE45 solver to solve the differential equation
[t,y] = ode45(eqn, tspan, y0);

% Plot the solution
plot(t, y(:,1))
xlabel('t')
ylabel('y(t)')

title('Solution to (2t-1)^2y'''' + (2t-1)y'' - 2y = 8t^2-2t+3')
```

Output:



Exercise Problems

1. $(D^3 + 2D^2 + D)y = x^2 e^{2x} + \sin^2 x \qquad y(2) = 0, y'(2) = -1$
2. $(D^2 - 2D + 4)y = e^x \cos x, \qquad y(1) = 0, y'(1) = -1$
3. $x^3 \frac{d^3y}{dx^3} + 3x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = x\log x\,, \quad y(2) = 0, y'(2) = -1$

Lab 7

Laplace Transformation

Problem: Find Laplace transform of a given function

Program

```
clc;
clear all;
syms t s a b  %representation of symbolic variables

f=input('Enter the function for which Laplace transform is required\n');
F=simplify(laplace(f));
fprintf('L[%s] = ', f);
disp(F)
%pretty(F) % display the result in a readable format
```

Problem: Find the Laplace transform of following functions:

1) e^t 2) $e^{-2t}(2\cos(5t) - \sin(5t))$ 3) $\mathrm{t}^3\sin(\mathrm{t})$ 4) $\mathrm{te}^{-2\mathrm{t}}\sin(4\mathrm{t})$

5) $\frac{cos(at)-cos(bt)}{t}$ 6) $\int_0^t \mathrm{t}\cos(at)\, dt$

Output:

```
1.  Enter the function for which Laplace transform is required
    exp(t)
    L[exp(t)] = 1/(s - 1)
2.  Enter the function for which Laplace transform is required
    exp(-2*t)*(2*cos(5*t)-sin(5*t))
    L[exp(-2*t)*(2*cos(5*t) - sin(5*t))] = (2*s - 1)/(s^2 + 4*s + 29)
3.  Enter the function for which Laplace transform is required
    t^3*sin(t)
    L[t^3*sin(t)] = (24*s*(s^2 - 1))/(s^2 + 1)^4
4.  Enter the function for which Laplace transform is required
    t*exp(-2*t)*sin(4*t)
    L[t*sin(4*t)*exp(-2*t)]= (4*(2*s + 4))/((s + 2)^2 + 16)^2
5.  Enter the function for which Laplace transform is required
    (cos(a*t)-cos(b*t))/t
    L[(cos(a*t) - cos(b*t))/t] = log(b^2/s^2 + 1)/2 - log(a^2/s^2 + 1)/2
6.  Enter the function for which Laplace transform is required
    int(t*cos(a*t),t,0,t)
    L[-(2*sin((a*t)/2)^2 - a*t*sin(a*t))/a^2] = -(a^2 - s^2)/(s*(a^2 + s^2)^2)
```

Exercise Problems

1) $sin(at)$	2) $cos(t)$	3) $\sinh(at)\sin(at)$	4) $\mathrm{t}^3\cosh(\mathrm{t})$
5) $3\sqrt{t} + \frac{4}{\sqrt{t}}$	6) $\frac{t-e^{-at}}{t}$	7) $\frac{2\sin(\mathrm{t})\sin(5\mathrm{t})}{\mathrm{t}}$	8) $\sin^2(2t+1)$
9) $e^{-4t}\int_0^t \mathrm{t}\sin(3\mathrm{t})dt$	10) $\int_0^t \sinh(at)\sin(at)\, dt$		

Lab 8

Laplace Transformation of Periodic function

Program:

```
clc;
clear all;
close all;
syms t s a b w E
f=input('Enter the periodic function\n');
T=input('Enter the period of the function\n');
F=(1/(1-exp(-s*T)))*(int(exp(-s*t)*f,t,0,T));
F1 = simplify(F);
fprintf('L[f(t)]  = ');
disp(F1)
pretty(F1) % display the result in a readable format
```

Problem: Find the Laplace transform of following functions:

1) E $\sin\left(\frac{\pi t}{w}\right)$, with $f(t+w)=f(t)$ 2) $E\,\sin(wt)$ with period π/w

Output:

```
1) Enter the periodic function
   E*sin(pi*t/w)
   Enter the period of the function
   w
   L[f(t)]  = -(E*w*pi*exp(-s*w)*(exp(s*w) + 1))/((exp(-s*w) - 1)*(pi^2 +
   s^2*w^2))

     E w pi exp(-s w) (exp(s w) + 1)
   - -------------------------------
                            2     2  2
      (exp(-s w) - 1) (pi   + s   w )
2) Enter the periodic function
   E*sin(w*t)
   Enter the period of the function
   w
   L[f(t)]  = -(E*(w*cos(w^2) + s*sin(w^2) - w*exp(s*w)))/((w^2 +
   s^2)*(exp(s*w) - 1))

               2           2
     E (w cos(w ) + s sin(w ) - w exp(s w))
   - --------------------------------------
              2    2
            (w  + s ) (exp(s w) - 1)
```

Laplace Transformation of piecewise Periodic function

Program to find Laplace transform of the function with period T of the form

$$f(t)=\begin{cases} f1(t) & \text{if} \quad T0 < t \le T1 \\ f2(t) & \text{if} \quad T1 \le t < T \end{cases}$$

Program:

```
clc;
clear all;
syms t s a b w E
T=input('Enter the period T of the function: \n');
T0=input('Enter the T0 value: \n'); %lower limit the function in first
subinterval
T1=input('Enter the T1 value: \n'); %upper limit the function in first
subinterval
fprintf('Enter the function in subinterval [%d,%s]: \n', T0, T1);
f1=input(' ');

fprintf('Enter the function in subinterval [%s,%s]: \n', T1, T);
f2=input(' ');

F=(1/(1-exp(-s*T)))*((int(exp(-s*t)*f1,t,T0,T1))+(int(exp(-s*t)*f2,t,T1,T)));
F1 = simplify(F);
fprintf('L[f(t)]  = ');
disp(F1)
%pretty(F1)
```

Problem: Find the Laplace transform of following functions:

1) $f(t) = \begin{cases} t, & 0 < t < a \\ 2a - t, & a < t < 2a \end{cases}$ where $f(t + a) = f(t)$

2) $f(t) = \begin{cases} E, & 0 < t < a/2 \\ -E, & a/2 < t < a \end{cases}$ where $f(t + a) = f(t)$

Output:

```
1) Enter the period T of the function:
   2*a
   Enter the T0 value:
   0
   Enter the T1 value:
   a
   Enter the function in subinterval [0,a]:
    t
   Enter the function in subinterval [a,2*a]:
    2*a-t
   L[f(t)]  = (exp(a*s) - 1)/(s^2*(exp(a*s) + 1))
2) Enter the period T of the function:
   a
   Enter the T0 value:
   0
   Enter the T1 value:
   a/2
   Enter the function in subinterval [0,a/2]:
    E
   Enter the function in subinterval [a/2,a]:
    -E
   L[f(t)]  = (E*(exp((a*s)/2) - 1))/(s*(exp((a*s)/2) + 1))
```

Laplace Transformation of Heaviside function

Program:

```
clc;
clear all;
syms t s a b
n=input('Enter the number of subintervals: ');
for i=1:n-1
    fprintf('Enter limit T(%d) of subinterval: ',i);
    T(i)=input('');
end

for i=1:n
    fprintf('Enter f%d(t) in subinterval [T(%d), T(%d)]: ',i,i-1,i);
    f(i)=input(' ');
end
sum=f(1);
for i=1:n-1
    k =(f(i+1)-f(i))*heaviside(t-T(i));
    sum=sum+k;
end
F = simplify(laplace(sum));
fprintf('L[f(t)] = ');
disp(F)
% pretty(F)
```

Problem: Find the Laplace transform of following functions:

1) $f(t) = \begin{cases} sint \;\; if \;\; 0 < t \le \pi/2 \\ cost \;\; if \;\;\;\;\;\; t > \pi/2 \end{cases}$

2) $f(t) = \begin{cases} \sin t \;\; if \;\; 0 < t < \pi \\ \sin 2t \;\; if \;\; \pi < t < 2\pi \\ \sin 3t \;\; if \;\;\;\;\;\; t > 2\pi \end{cases}$

Output:

```
1) Enter the number of subintervals: 2
   Enter limit T(1) of subinterval: pi/2
   Enter f1(t) in subinterval [T(0), T(1)]:  sin(t)
   Enter f2(t) in subinterval [T(1), T(2)]:  cos(t)
   L[f(t)] = 1/(s^2 + 1) - exp(-(pi*s)/2)*(s/(s^2 + 1) + 1/(s^2 + 1))
2) Enter the number of subintervals: 3
   Enter limit T(1) of subinterval: pi
   Enter limit T(2) of subinterval: 2*pi
   Enter f1(t) in subinterval [T(0), T(1)]:  sin(t)
   Enter f2(t) in subinterval [T(1), T(2)]:  sin(2*t)
   Enter f3(t) in subinterval [T(2), T(3)]:  sin(3*t)
   L[f(t)] = 1/(s^2 + 1) - exp(-2*pi*s)*(2/(s^2 + 4) - 3/(s^2 + 9)) +
   exp(-pi*s)*(1/(s^2 + 1) + 2/(s^2 + 4))
```

Exercise Problems

Find the Laplace transform of the following functions by expressing it in terms of Heaviside function.

1) $f(t) = \begin{cases} 5+t, & 0 < t < 2, \\ t^2 + 2t - 2, & t > 2. \end{cases}$

2) $f(t) = \begin{cases} t^2, & 0 < t < 2, \\ t-1, & 2 < t < 4, \\ 7, & t > 4. \end{cases}$

Lab 9

Inverse Laplace Transformation

Program:

```
clc;
clear all;
syms t s a b  %representation of symbolic variables
f=input('Enter the function for which inverse Laplace transform is
required\n');
z=simplify(ilaplace(f));
fprintf('Laplace inverse of [%s] = ', f);
disp(z)
%pretty(z) % display the result in a readable format
```

Problem 01: Find the Inverse Laplace transform of following functions:

1) $\frac{1}{s-a}$ 2) $\frac{e^{-\pi s}}{s^2+1}$ 3) $\frac{1}{(s-4)^2}$ 4) $\log\left(1-\frac{a^2}{s^2}\right)$

Output:

1)
```
Enter the function for which inverse Laplace transform is required
1/(s-a)
Laplace inverse of [-1/(a - s)] = exp(a*t)
```
2)
```
Enter the function for which inverse Laplace transform is required
exp(-pi*s)/((s^2)+1)
Laplace inverse of [exp(-s*pi)/(s^2 + 1)] = -sin(t)*heaviside(t - pi)
```
3)
```
Enter the function for which inverse Laplace transform is required
1/(s-4)^2
Laplace inverse of [1/(s - 4)^2] = t*exp(4*t)
```
4)
```
Enter the function for which inverse Laplace transform is required
log(1-(a^2/s^2))
Laplace inverse of [log(1 - a^2/s^2)] = -(2*(cosh(a*t) - 1))/t
```

Exercise Problems

Find the Inverse Laplace transform of the following functions:

1) $\frac{s}{s^2+a^2}$ 2) $\frac{3(s^2-1)^2}{2s^5}$ 3) $\frac{5s+3}{(s-1)(s^2+2s+5)}$ 4) $\log\left(\frac{s+a}{s+b}\right)$

Lab 10

Convolution theorem in Laplace transform

Program:

```
clc
clear all
syms s t u a b
F = input('Enter function F(s): ');
G = input('Enter function G(s): ');
L=simplify(F*G);
f(t) = ilaplace(F);
g(t) = ilaplace(G);
R=simplify(int(f(u)*g(t-u),u,0,t));
fprintf('Inverse Laplace Transform of %s  =\n', L);
disp(R)
%pretty(R)
```

Problem 01: Using convolution theorem obtain the inverse Laplace transform of $f(x) = \frac{1}{s(s^2+a^2)}$

Output:

```
Enter function F(s): 1/s
Enter function G(s): 1/(s^2+a^2)

Inverse Laplace Transform of 1/(s*(a^2 + s^2))  =
-(cos(a*t) - 1)/a^2
```

Exercise Problems

Find the Inverse Laplace transform of the following functions using Convolution theorem:

1) $\frac{s}{(s^2+a^2)^2}$

2) $\frac{s^2}{(s^2+a^2)(s^2+b^2)}$

3) $\frac{1}{(s-1)(s^2+1)}$

4) $\frac{s+2}{(s^2+4s+5\,)^2}$

5) $\frac{1}{s^2\,(s+1)^2}$

6) $\frac{4s+5}{(s-1)^2(s+2)}$